LTI's with switched delays

and applications to WCNs

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BALCON-HYCON2 meeting

Belgrade, July 2013.

Joint work with M-D DiBenedetto, A. D'Innocenzo (DEWS, L'Aquila)

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Wireless control networks:
a paradigm shift

HART COMMUNICATION FOUNDATION

Weaknesses of Wired Control Networks

Wires are expensive

Lack of flexibility

Restricted control architectures

Opportunities with
Wireless Control Networks (WCN)

Lower costs, easier installation

Broadens scope of sensing and control

Compositionality

Runtime adaptation

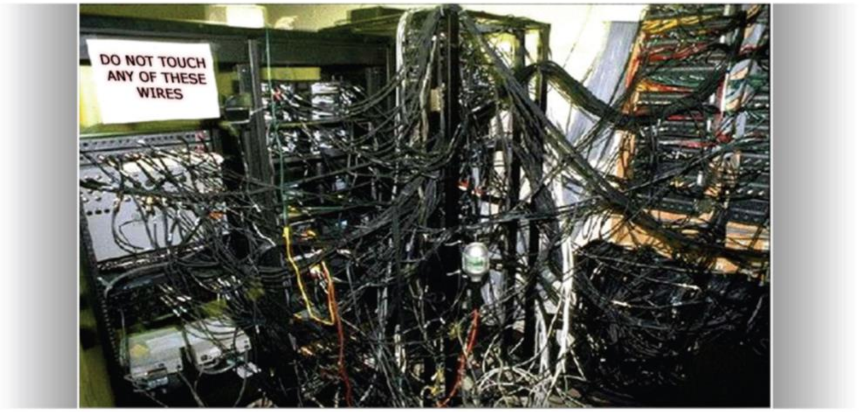
Technical Challenges with WCN

Modeling

Analysis

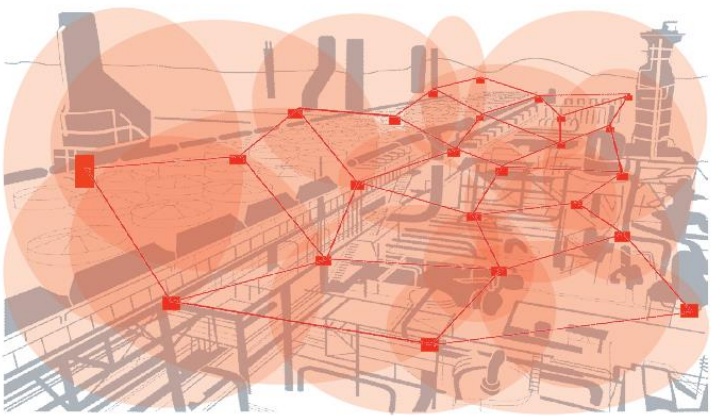
(Co-)Design

Robustness



Courtesy of Honeywell

WirelessHART



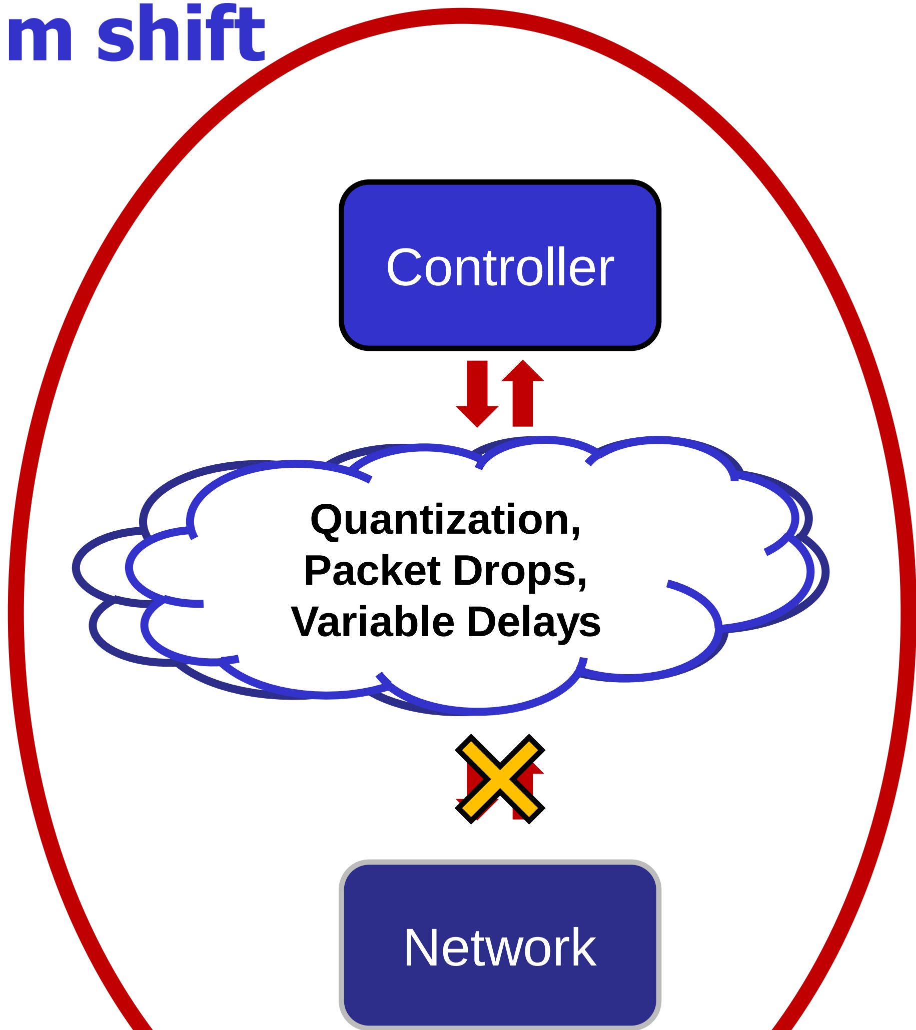
Wireless control networks:
a paradigm shift

Previous approaches consider network non-idealities as a separate communication problem (quantization, packet drops, variable delays) [Andersson:CDC05, Antsaklis:TAC04, Heemels:TAC10, espanha:ProcIEEE07, Murray:SMTNS06]

➔**Potential loss of optimality**

Long term goal:

- Mathematical model of time-triggered communication protocols
- Relate network non-idealities to network parameters: topology, transmission power, scheduling, routing and incorporte them in the control process
- Optimize the overall process in an integrated way



ISA100Wireless

WirelessHART

Outline

LTIs with switched delays

- Analysis
- Design: Hardness and theoretical limits
- A structure theorem

Conclusion and perspectives

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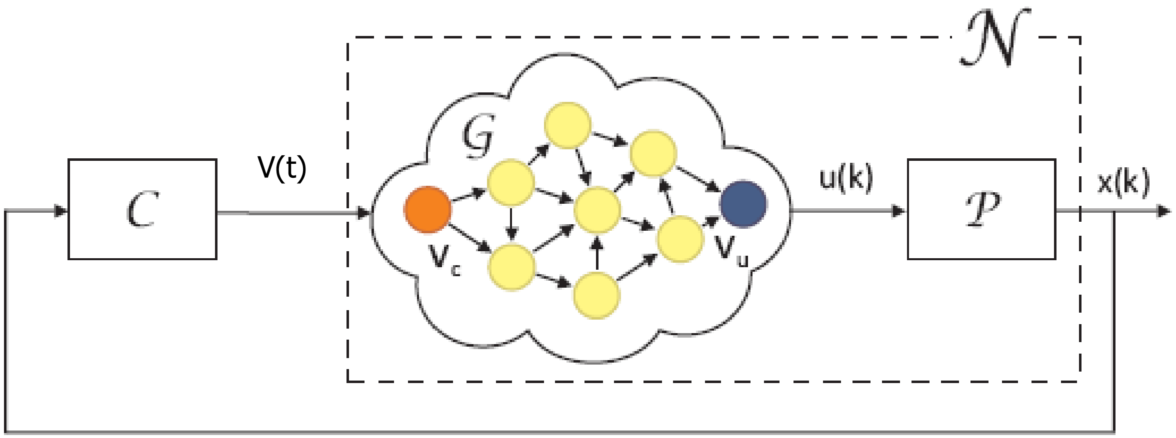
Conclusion and perspectives

Design and modeling of the multihop control network

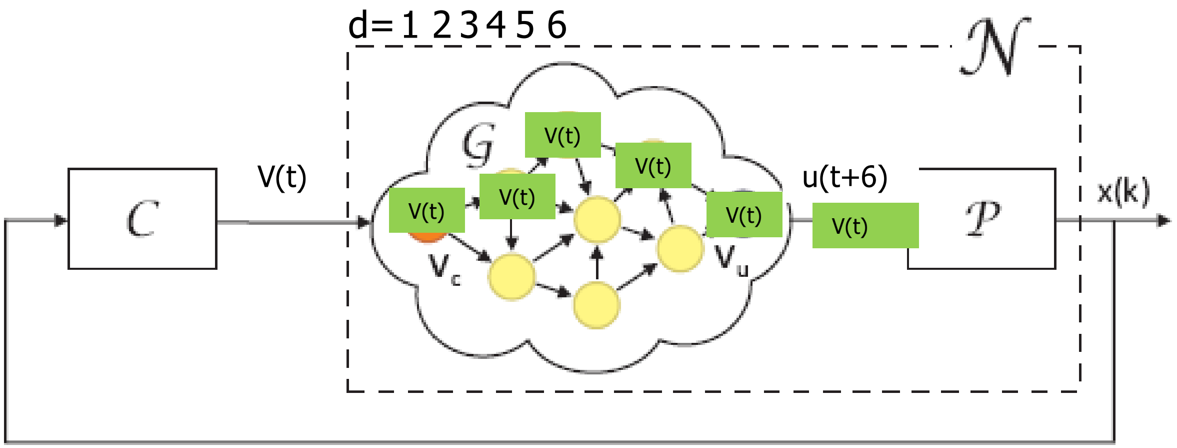


Centralized Controller, Relay Network, no data processing, acyclic graph

[Alur, D'Innocenzo, Johansson, Pappas, Weiss, IEEE-TAC 2011]

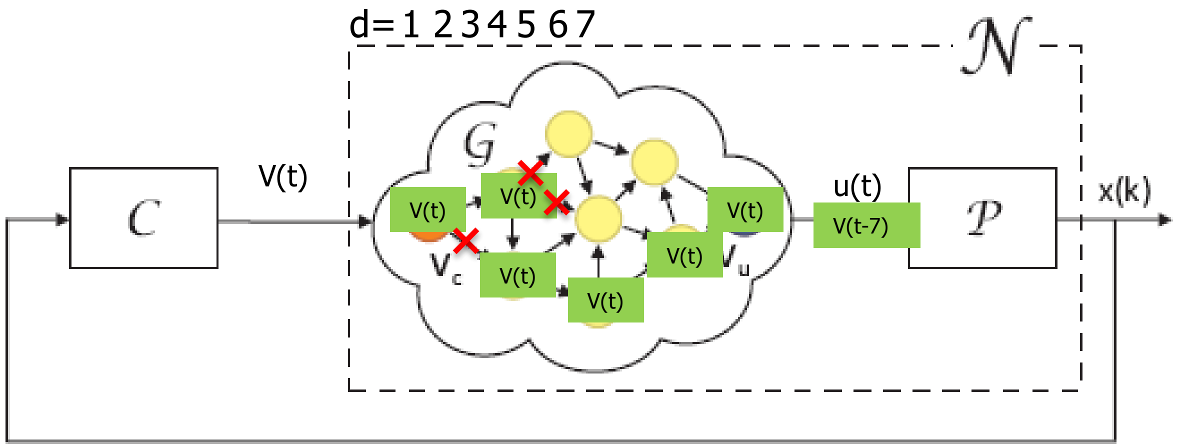
(for other control strategies, see [Pajic, Sundaram, Pappas, Mangharam, IEEE-TAC 2011], [D'Innocenzo, Di Benedetto, Serra, IEEE-TAC, 2013])

Design and modeling of the multihop control network



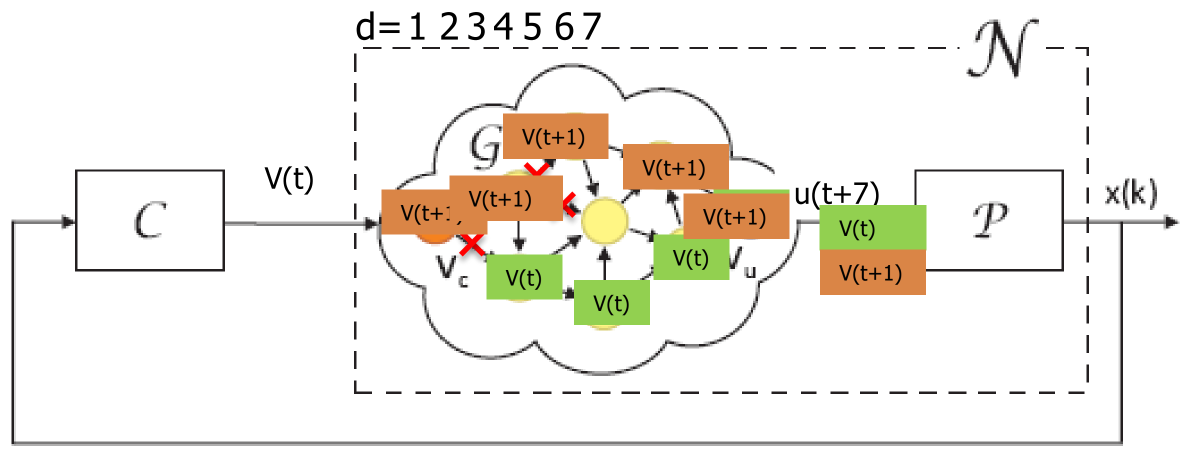
$$x(t+1) = Ax + Bu(t-d)$$

How to model failures?
LTIs with switched delays



$$x(t+1) = Ax + Bu(t - d_2)$$

How to model failures? LTIs with switched delays



$$x(t+1) = Ax(t) + Bu(v(t-d_{max}:t), \sigma(t-d_{max}:t))$$

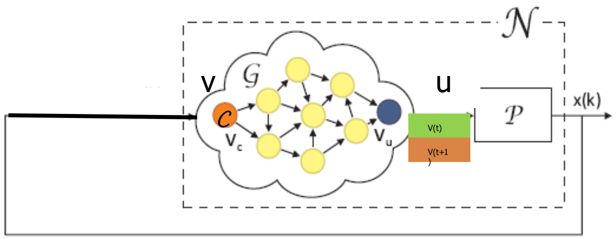
$$x(t+1) = Ax + Bu(t-d_2)$$

$D = \{d_1, \ldots, d_{|D|}\}$ is the set of possible delays

d_{max} Is the maximal delay

LTIs with switched delays
The linear controller

Delay dependent controller

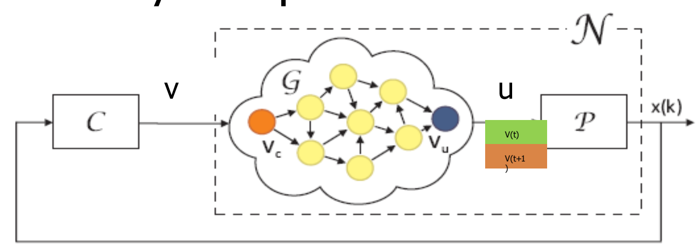


$$v(t) = K(d)\tilde{u}(t)$$

$$\tilde{u}(t) =$$

$$(x(t), u_1(t), u_2(t), \ldots, u_{d_{max}}(t))$$

Delay independent controller



$$v(t) = K\tilde{v}(t),$$

$$\tilde{v}(t) =$$

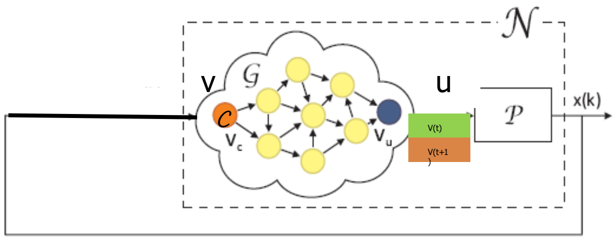
$$(x(t), v(t - d_{max}), \ldots, v(t-1)))$$

$$x(t+1) = Ax(t) + Bu(v(t-d_{max} : t), \sigma(t-d_{max} : t))$$

$$u_{\cdot}(t) = \sum_{t' < t : t' + \sigma(t') = t} v(t')$$

LTIs with switched delays
The linear controller

Delay dependent controller



$$v(t) = K(d)\tilde{u}(t)$$

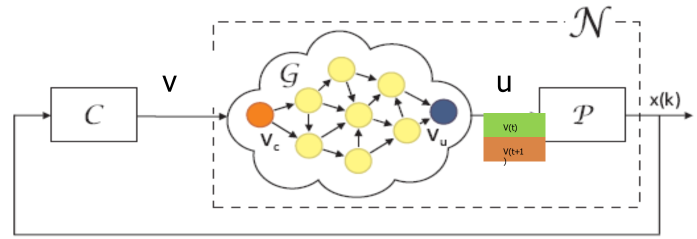
$$\tilde{u}(t) =$$

$$(x(t), u_1(t), u_2(t), \ldots, u_{d_{max}}(t))$$

$$\Sigma = \left\{ \begin{pmatrix} A & B & 0 & \ldots & 0 \\ 0 & 0 & I & \ldots & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \ldots & I \\ 0 & 0 & 0 & \ldots & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \vdots \\ K(d) \\ \vdots \\ 0 \end{pmatrix} \right\}$$

[Hetel Daafouz Iung 07]
[Weiss et al. 09]

Delay independent controller



$$v(t) = K\tilde{v}(t),$$

$$\tilde{v}(t) =$$

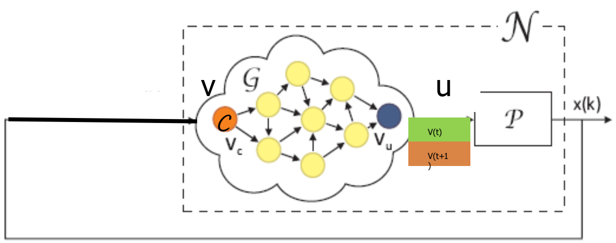
$$(x(t), v(t - d_{max}), \ldots, v(t-1)))$$

$$\Sigma = \left\{ \begin{pmatrix} A & 0 & \boxed{B} & ..\boxed{B} & 0 \\ 0 & 0 & I & \ldots & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ K_0 & K_1 & K_2 & \ldots & K_{d_{max}} \end{pmatrix} \right\}$$

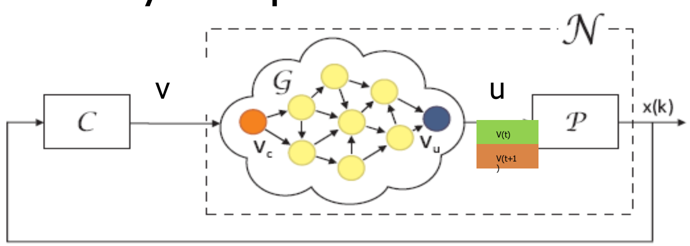
[J. D'Innocenzo Di Benedetto 12]

LTIs with switched delays

Delay dependent controller



Delay independent controller



- Corollary:

  For both models there is a PTAS for the stability question:

  for any required accuracy, there is a polynomial-time algorithm for checking stability up to this accuracy

i.e. we have necessary and sufficient conditions for stability

Previous sufficient conditions in [Hetel Daafouz Iung 07, Zhang Shi Basin 08]

- However:

  Theorem: the very stability problem is NP-hard

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Conclusion and perspectives

Design of LTIs with switched delays the delay-dependent case

➔ Question: how to algorithmically decide if stabilization is possible, even in this simplest case?

- Delay dependent:
  Exemple

$$A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \end{pmatrix}^T$$

$$D = \{0, 1\}, \quad \sigma(t) = t \bmod 2$$

$$x_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$x_1 = A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x_2 = A^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + Bv(1) + Bv(2) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ v(1) + v(2) \end{pmatrix}$$

➔ uncontrollable

Design of LTIs with switched delays

- Theorem: for n=m=1, there is an explicit formula for a delay-dependent controller that achieves finite time stabilization.

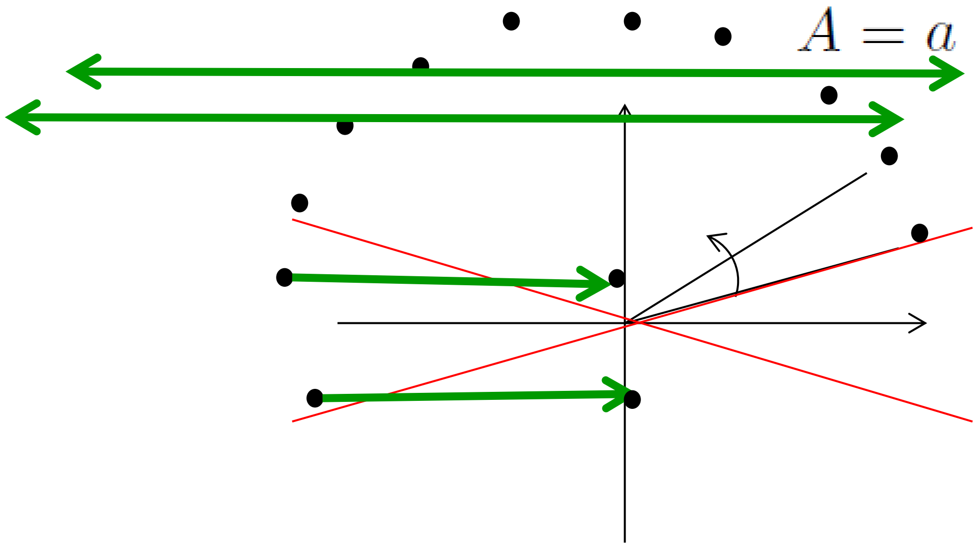
  (based on a generalization of the Ackermann formula for delayed LTI)

Design of LTIs with switched delays the delay-independent case

A linear controller is not always sufficient:

Exemple: a 2D system with two possible delays

$$A = a\begin{pmatrix} cos(\alpha) & sin(\alpha) \\ -sin(\alpha) & cos(alpha) \end{pmatrix}$$



$$b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$D = \{0, 1\}$$

- Theorem: For the above system, there exist values of the parameters such that no linear controller can stabilize the system, but a nonlinear bang-bang controller does the job.

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A structure theorem
For the delay dependent controller

Suppose we know the sequence of delays

- Theorem: A sequence of delays is uncontrollable if A can be put in a p-block permutation form, with b having a zero block in the same basis

$$A = \left(\begin{array}{c|c|c} & X & \\ \hline & & X \\ \hline X & & \end{array}\right) \quad b = \left(\begin{array}{c} X \\ \hline \\ \hline \\ \end{array}\right)$$

and

$$\forall t,\ t + d(t) \neq k \mod p$$

- Theorem: There is a polynomial time algorithm that decides whether such an adversary strategy is possible

- Conjecture: An LTI with switching delays is uncontrollable if and only if the situation in the preceding theorem (in fact, a slightly more general) holds

Design of LTIs with switched delays the delay-dependent case

➔ Question: how to algorithmically decide if stabilization is possible, even in this simplest case?

- Delay dependent:
  Exemple

$$A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \end{pmatrix}^T$$

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➔ uncontrollable

Outline

- A tool for switching systems analysis
- LTIs with switched delays
- Conclusion and perspectives

Outline

Desired characteristics or descriptive words of well-designed and engineered Cyber-Physical Systems include: coordinated, distributed, connected, heterogeneous, robust and responsive, providing new capability, adaptability, resiliency, safety, security,

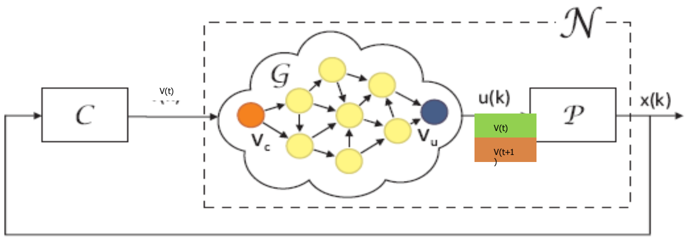
. Very difficult challenges are posed for control of CPS, however, due to a variety of factors such as very broad time and length scales, the presence of network communication and delays,

- Indeed there are many open questions!
  - What are the best control schemes?
  - How to design a (delay dependent/independent) controller?
  - Is the one dimensional case decidable with d-i controllers?
  - What about the expected (vs. Worst-case) stability?
  - …

Thanks!

Questions?

Ads



The JSR Toolbox: http://www.mathworks.com/matlabcentral/fileexchange/33202-the-jsr-toolbox

References:
http://perso.uclouvain.be/raphael.jungers/

Joint work with
M-D di Benedetto (DEWS), A. D'innocenzo (DEWS)

Positions available in my team