# **Adversary** Control Strategy for **Cyberphysical Networked Systems**

# **Efstathios Kontouras and Anthony Tzes**

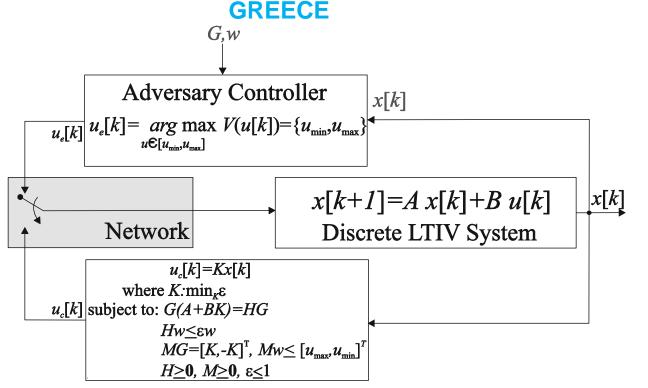
**University of Patras** 

**Electrical & Computer Engineering Department** 





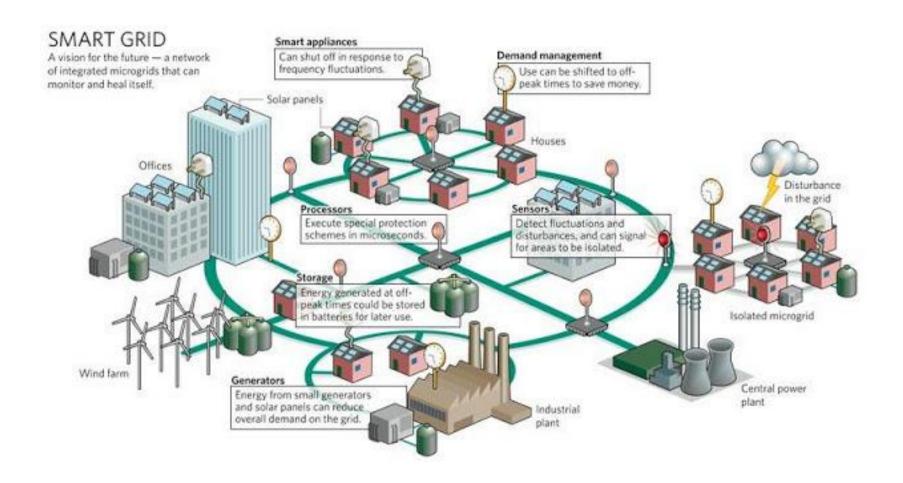




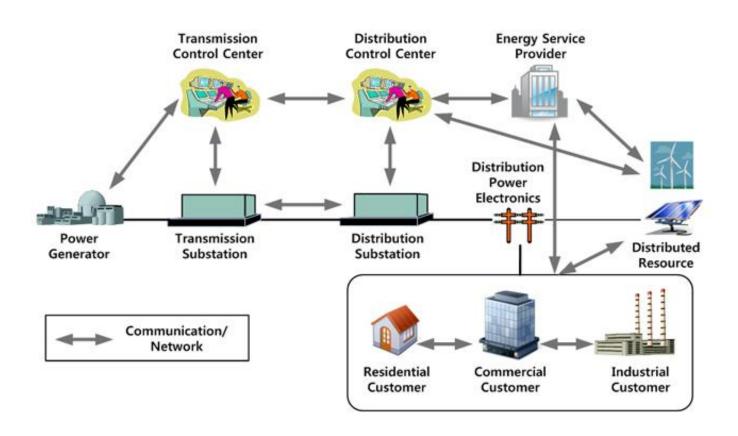




# Motivation: Adversary Control for Frequency Instability in Distributed Power Systems



# Adversary Attacks at Communications Network of the «Transmission and Distribution Control Center»



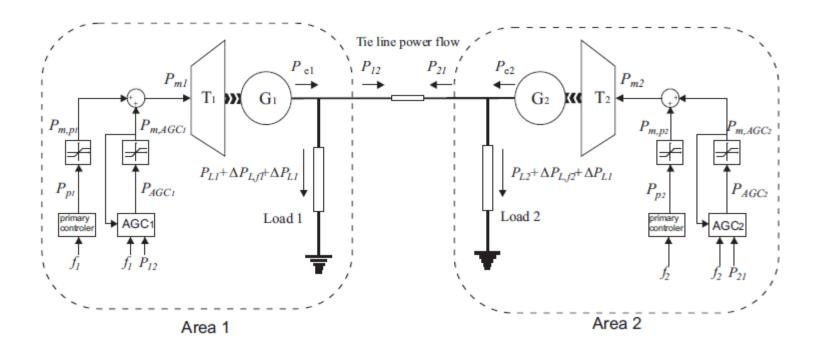


Fig. 1. Two-Area Power System with AGC Lygeros et al, ACC09

$$\Delta \dot{f}_{1} = \frac{f_{0}}{2H_{1}S_{B_{1}}} \left( \Delta P_{m,p_{1}} + \Delta P_{m,AGC_{1}} - \frac{1}{D_{l_{1}}} \Delta f_{1} - P_{T} \sin(\Delta \phi + \phi_{0}) + P_{0_{12}} \right),$$

$$\Delta \dot{f}_{2} = \frac{f_{0}}{2H_{2}S_{B_{2}}} \left( \Delta P_{m,p_{2}} + \mathbf{u} - \frac{1}{D_{l_{2}}} \Delta f_{2} + P_{T} \sin(\Delta \phi + \phi_{0}) - P_{0_{12}} \right),$$

$$\Delta \dot{\phi} = 2\pi (\Delta f_{1} - \Delta f_{2}),$$

$$\Delta \dot{P}_{AGC_{1}} = \left( \frac{1}{D_{l_{1}}} \frac{C_{p_{1}} f_{0}}{2S_{1}H_{1}S_{B_{1}}} - \frac{1}{S_{1}} \frac{1}{T_{N_{1}}} \right) \Delta f_{1}$$

$$- \frac{C_{p_{1}} f_{0}}{2S_{1}H_{1}S_{B_{1}}} \Delta P_{m,p_{1}} - \frac{C_{p_{1}} f_{0}}{2S_{1}H_{1}S_{B_{1}}} \Delta P_{m,AGC_{1}}$$

$$- \left( \frac{1}{T_{N_{1}}} - \frac{C_{p_{1}} f_{0}}{2S_{1}H_{1}S_{B_{1}}} \right) (P_{T} \sin(\Delta \phi + \phi_{0}) - P_{0_{12}})$$

$$- 2\pi C_{p_{1}} P_{T} (\Delta f_{1} - \Delta f_{2}) \cos(\Delta \phi + \phi_{0}) - \frac{K_{a_{1}}}{T_{N_{1}}} p_{1}. \tag{1}$$

$$\dot{x} = f(x, w) + g(x, w)u$$

#### *u*: disturbance

$$\Delta P_{p_i} = -1/S_i$$
 for i=1,2

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$$\dot{x} = f(x, w) + g(x, w)u$$

$S_{B_i}$	$f_0$	$D_{l_i}$	$S_i$	$C_{p_i}$	$T_{N_i}$
10 <i>GW</i>	50 <i>н</i> z	$\frac{1}{200}MW/Hz$	0.002Hz/MW	0.1	30
$\Delta P_{AGC_i}^{max}$	$\Delta P_{AGC_i}^{min}$	$\Delta P_{p_i}^{max}$	$\Delta P_{p_i}^{min}$	$P_T$	Ka
350 <i>мŵ</i>	-350 <i>мw</i>	75 <i>MW</i>	-75 MW	1000 <i>mw</i>	100

TABLE I

PARAMETER VALUES FOR THE TWO AREA POWER SYSTEM

$$\Delta P_{m,p_{i}} = \begin{cases} \Delta P_{p_{i}}^{min} & \text{if } \Delta P_{p_{i}} \leq \Delta P_{p_{i}}^{min} \\ \Delta P_{p_{i}} & \text{if } \Delta P_{p_{i}}^{min} < \Delta P_{p_{i}} < \Delta P_{p_{i}}^{max} \end{cases}$$

$$\Delta P_{max} \text{ if } \Delta P_{p_{i}}^{min} \leq \Delta P_{p_{i}}^{min}$$

$$\Delta P_{p_{i}}^{max} \text{ if } \Delta P_{p_{i}} \geq \Delta P_{p_{i}}^{max}$$

$$\Delta P_{p_{i}}^{max} \text{ Otherwise}$$

$$\text{Load shedding}$$

$$\text{Generator tripping}$$

$$\Delta P_{m,AGC_1} = \begin{cases} \Delta P_{AGC_1}^{min} & \text{if } \Delta P_{AGC_1} \leq \Delta P_{AGC_1}^{min} & \text{:} \\ \Delta P_{AGC_1} & \text{if } \Delta P_{AGC_1}^{min} < \Delta P_{AGC_1} < \Delta P_{AGC_1}^{max} & \text{:} \\ \Delta P_{AGC_1}^{max} & \text{if } \Delta P_{AGC_1} \geq \Delta P_{AGC_1}^{max} & \text{System Blackout} \end{cases}$$

$$p_{1} = \begin{cases} 0 & \text{if } \Delta P_{AGC_{1}}^{min} < \Delta P_{AGC_{1}} < \Delta P_{AGC_{1}}^{max} \\ \Delta P_{AGC_{1}} - \Delta P_{m,AGC_{1}} & \text{else} \end{cases}$$
$$\Delta P_{12} = P_{T} \sin(\Delta \phi + \phi_{0}) - P_{0_{12}}$$

where  $\phi_0$  is the angle difference that corresponds to the scheduled transferred power i.e.  $P_{0_{12}} = P_T \sin(\phi_0)$ 

Adversary Control: Computation of optimal u, such that the system's state vector to be found outside of its safe operating region (relative to its frequency)

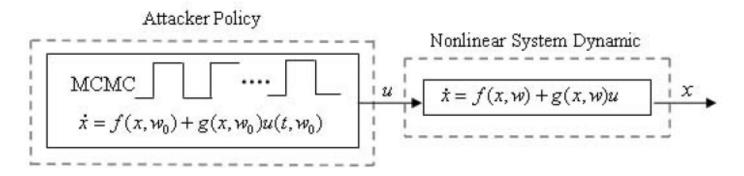
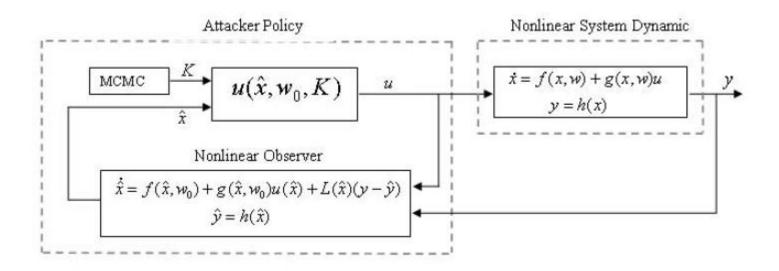


Fig. 2. Block diagram of open loop policy for the attacker



$$S: x[k+1] = Ax[k] + Bu[k], \quad x[0] = x_0,$$

State

$$\mathcal{R}(G, w) = \{ x \in \mathbb{R}^n : Gx \le w \},\$$

Constraints

$$u = Kx$$

$$Q(K, u_{\min}, u_{\max}) = \left\{ x \in \mathbb{R}^n : \begin{bmatrix} K \\ -K \end{bmatrix} x \le \begin{bmatrix} u_{\max} \\ -u_{\min} \end{bmatrix} \right\}$$

Optimal Feedback

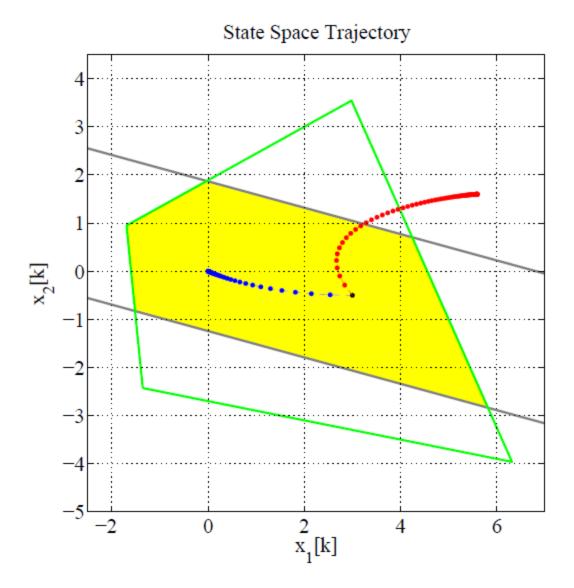
1) Ensure invariance of set  $\mathcal{J} = \mathcal{R} \cap \mathcal{Q}$ 

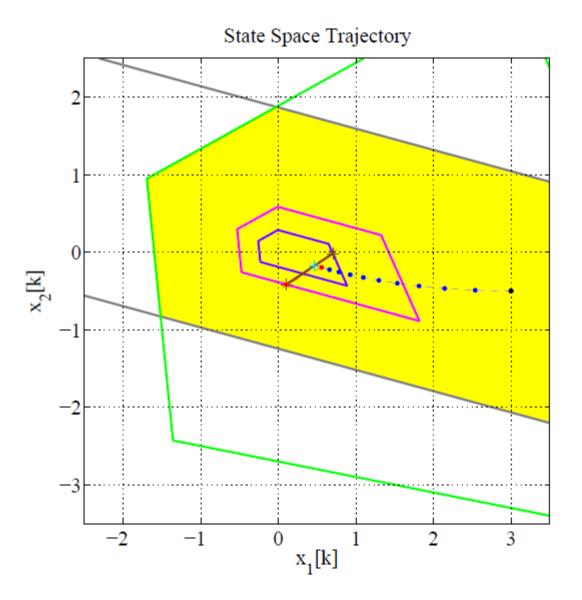
Control Objective

2) Drive *x* to 0 at the fastest possible rate

Adversary
Control
Objective

Compute u: such that state vector x exits as fast as possible from set  $\mathcal{J}$ 



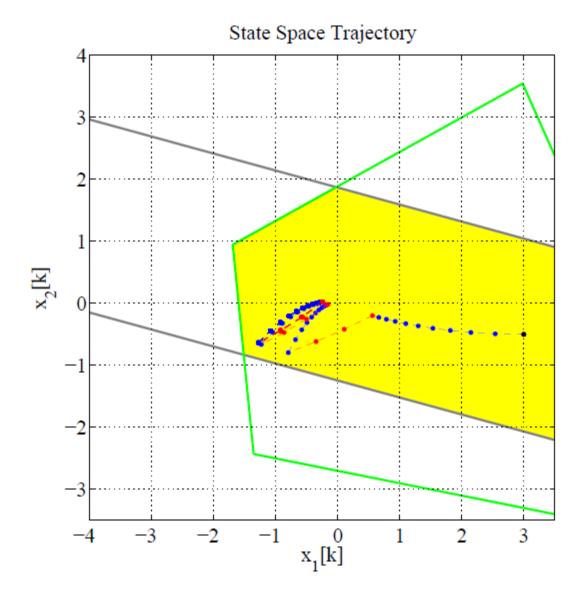


V(x) isocontours, and adversary control command selection

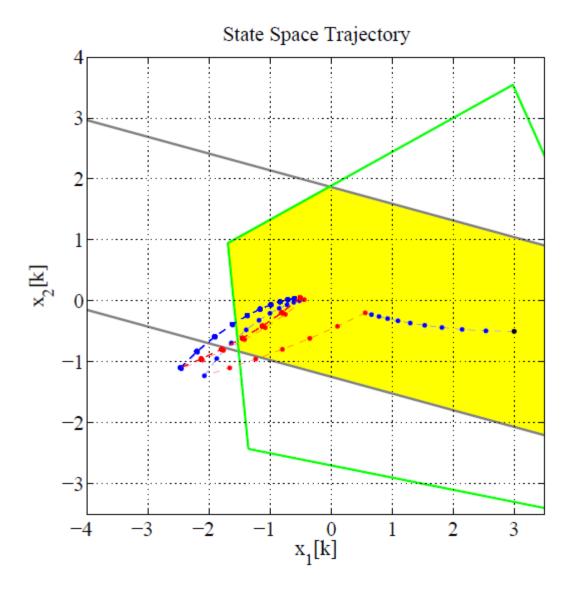
# Optimal Feedback Control

$$\begin{array}{ll} \min_{\varepsilon,K,H,M}\{\varepsilon\}, & \text{subject to} \\ G(A+BK) & = & HG \\ Hw & \leq & \varepsilon w, \ H \in \mathbb{R}^{m \times m} \\ MG & = & K, \quad Mw \leq \rho, \quad M \in \mathbb{R}^{2 \times m} \\ \varepsilon & \leq & 1, \ H \geq \mathbb{O}, M \geq \mathbb{O} \end{array}$$

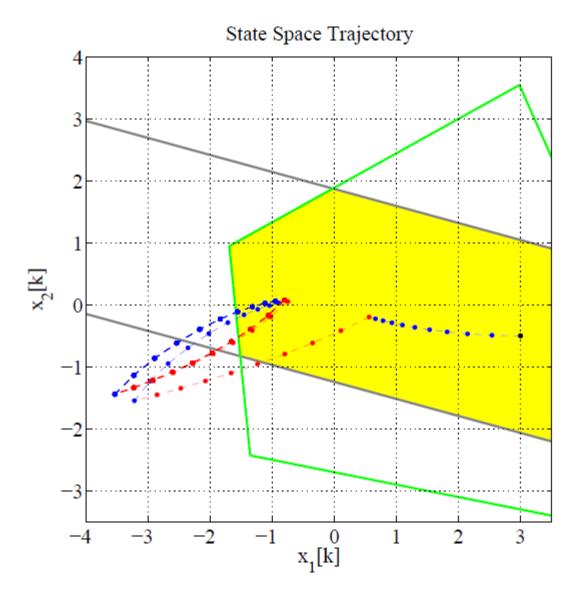
Adversary Control	$V(x) = \max_{i=1,2,\dots,m} \left\{ \max \left\{ \frac{(G'x)_i}{w'_i}, 0 \right\} \right\}$		
	$(G'x)_i$ and $w'_i$ denote the <i>i</i> -th element of vectors		
	$\lceil G \rceil \qquad \lceil w \rceil$		
	$G'x = \begin{bmatrix} G \\ K \\ -K \end{bmatrix} \text{ and } w' = \begin{bmatrix} w \\ u_{\text{max}} \\ -u_{\text{min}} \end{bmatrix}$		
	$\lfloor -K \rfloor$ $\lfloor -u_{\min} \rfloor$		
	$u_e[k] = \underset{-}{\operatorname{argmax}} V(u[k])$		
	$u \in [u_{\min}, u_{\max}]$		
	$\int u_{\min},  \text{if } V(u_{\min}) \ge V(u_{\max})$		
	$= \begin{cases} u_{\min}, & \text{if } V(u_{\min}) \ge V(u_{\max}) \\ u_{\max}, & \text{if } V(u_{\min}) < V(u_{\max}) \end{cases} $		



State vector behaviour for  $n_c = 10$  and  $n_e = 3$ 



State vector behaviour for  $n_c = 10$  and  $n_e = 6$ 



State vector behaviour for  $n_c = 10$  and  $n_e = 9$ 

# **Conclusions**

### If the Adversary Controller has access to:

- 1) the description of the system's dynamics
- 2) the state vector measurements
- 3) the feedback control policy
- 4) the system's state and input constraints

Then an optimum adversarial strategy was provided

#### Issues of future interest

- 1) State vector measurement uncertainties may be used in a game-approach (between feedback and adversary control)
- 2) System Parametric Uncertainty
- 3) MIMO systems
- 4) Nonlinear systems